Fuzzy Observer-Based Consensus Tracking Control for Fractional-Order Multi-Agent Systems Under Cyber-Attacks and Its Application to Electronic Circuits

G. Narayanan, M. Syed Ali, Quanxin Zhu, *Senior Member, IEEE*, Bandana Priya, and Ganesh Kumar Thakur

***Abstract*—Consensus control of multi-agent systems (MASs) has applications in various domains. As MASs work in networked environments, their security control becomes critically desirable in response to cyber-attacks. In this paper, the observer-based consensus tracking control problem is investigated for a class of Takagi-Sugeno fuzzy fractional-order multi-agent systems (FOMASs) under cyber-attacks. The malicious cyber attacks can impact the security of topologies of the communication networks of both controllers and observers. To estimate unmeasurable system states, a fuzzy observer is built. It is found that the topology of contact for observer states may be different from that of the feedback signals. A novel mathematical model for T-S fuzzy FOMASs with cyber-attacks is proposed. By using algebraic graph theory, Lyapunov functional, and fractional calculus theory, a distributed feed-back controller is developed for each agent, which guarantee the secure performance of tracking consensus error and observer error. Finally, two numerical examples demonstrate the effectiveness of the suggested control scheme, and the controller design for electronic network circuits shows the applicability of the proposed theoretical results. Simulations results for different differential-orders and coupling strength scenarios are given.**

***Index Terms*—Cyber-attacks, distributed control, fractional-order, multi-agent systems, takagi-sugeno model.**

Manuscript received 27 August 2022; accepted 19 October 2022. Date of publication 31 October 2022; date of current version 23 February 2023. The work of G. Narayanan was supported by the Center for Nonlinear Systems, Chennai Institute of Technology, India, under Grant CIT/CNS/2022/RD/003. This work was supported in part by the National Natural Science Foundation of China under Grant 62173139 and in part by the Science and Technology Innovation Program of Hunan Province under Grant 2021RC4030. Recommended for acceptance by Dr. Xuelong Li. *(Corresponding author: Quanxin Zhu.)*

G. Narayanan is with the Center for Computational Modeling, Chennai Institute of Technology, Chennai 600069, India (e-mail: narayanantvu@gmail.com).

M. Syed Ali is with the Department of Mathematics, Thiruvalluvar University, Vellore 632115, India (e-mail: syedgru@gmail.com).

Quanxin Zhu is with the School of Mathematics and Statistics, Hunan Normal University, Changsha 410081, China, and also with the Key Laboratory of Control and Optimization of Complex Systems, College of Hunan Province, Hunan Normal University, Changsha 410081, China (e-mail: zqx22@126.com).

Bandana Priya is with the Department of Applied Science, G L Bajaj institute of Technology and Management, Greater Noida 201306, India (e-mail: bandanapriyathakur@gmail.com).

Ganesh Kumar Thakur is with the ABES Engineering College, Ghaziabad 201009, India (e-mail: meetgangesh@gmail.com).

Digital Object Identifier 10.1109/TNSE.2022.3217618

I. Introduction

The distributed coordinated control of MASs have gained wider and wider attention due to its potential applications in several disciplines such as robotics [1], aircraft control [2], unmanned air vehicles [3], smart grids [4], and sensor networks [5]. We should note that much of the existing research on MASs consensus focuses on integer-order dynamics ([6]–[9]). The importance of dealing with fractional-order derivatives is the involvement of memory and hereditary properties that gives a more realistic way to fractional-order models ([10], [11]). Due to the memory effect, the non-integer models integrate all previous information from the past that makes it to predict and translate the fractional-order models more accurately. It has been discovered, in particular, that fractional-order systems, which are acknowledged as a major advance over integer-order systems, could be applied in a growing number of engineering application fields ([12]–[14]). Despite the fact that there are many studies on the consensus of MASs in integer-order case, there are few results on FOMASs ([15]–[19]). Compared with the results of MASs in integer-order case, the consensus problem of FOMASs is relatively few, which has the potential research value due to the memory of FOMASs. Because of practical constraints, some agents partial information may be unmeasurable. Thus, for the consensus tracking problems, agent output measurements are observed, and different techniques of observer-based control are studied ([20]–[27]). Compared with the published works in the literature, the obtained criteria improve the previous works. Therefore, it is of the great significance to study the observer-based control for the engineering application scopes of MASs.

A cyber-physical system (CPS) is an intelligence system consist of processing, communication, and control with both physical and cyber components. As established in ([28]–[31]), security concerns for CPSs differ from those in typical control systems because cyber-attacks in the cyber layer can be extended to the physical layer. With the advent of network information and broad spatial distributed systems, MASs which can be considered a subset of CPSs, are becoming vulnerable to cyber-attacks. In actuality, the network is very vulnerable to malicious signal attacks as a result of its openness and shareability ([32]–[37]). Thus, our results significance improve from former works.

The T-S fuzzy model is well known as a powerful tool for dealing with the leader-follower consensus in achieve MASs

([38]–[43]). Due to their significant usage of communication technologies, MASs are frequently exposed to various cyber-attacks. T-S fuzzy networked systems, which can be considered a class of CPSs, have been vulnerable to cyber-attacks as network information technology and large-scale spatial distributed systems have advanced. These attacks could have a significant impact on tracking performance ([44]–[46]). Implementing performance distributed secure control techniques for FOMASs under attack remains a challenging and significant concern. To the best of our knowledge, no one has explored the observer-based consensus tracking problem of T-S fuzzy FOMASs with cyber-attacks, and it is, therefore, beneficial to further develop new techniques dealing. This motivates our study.

As narrated above, we focused on the T-S fuzzy observer-based consensus tracking control of FOMASs under cyber-attack. The main contributions are:

(i) The secure consensus criteria is derived for FOMASs via T-S fuzzy approach under a cyber-attack scenario.

(ii) Compared with the existing results for MAS under cyber-attacks ([32]–[34]), under malicious attacks, the security control analysis of both controllers and observers communication networks is unrelated, and over the duration of the attack in this study, these two topologies can change.

(iii) We developed a useful technique for determining the coupling strengths and feedback gain matrices for the controllers and observers.

(iv) We developed two optimization problems that solved sufficient criteria to achieve consensus tracking.

(v) Finally, numerical simulations show that the suggested observer-based control scheme is used to the consensus tracking of a tunnel diode network circuit.

Notations: Let $\mathscr{N}$ is the natural number; Real numbers, and $n \times 1$ real (complex) column vectors are referred to in $\mathscr{R}$, and $\mathscr{R}^n(\mathscr{C}^n)$ respectively. 'T' denotes the matrix transposition. $\mathcal{I}_n$ represent identity matrix. $\lambda(\cdot)$ the eigenvalue of a matrix. $\otimes$ stands for the Kronecker product.

II. System Description and Preliminaries

A. Algebraic Graph Theory

When each agents is regarded a node, the FOMASs have $\mathcal{M}$ followers, and a single node can be represented as a directed graph, where $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$, where $\mathscr{V} = \{1, 2, \ldots, \mathcal{M}+1\}$ is the node set, $\mathscr{E} \subseteq \{(p,q), p, q \in \mathscr{V}\}$ is the edge set, and $\mathscr{A} = [a_{pq}] \in \mathscr{R}^{(\mathcal{M}+1)\times(\mathcal{M}+1)}$, which is called adjacent matrix of $\mathscr{G}$ with non-negative elements, where if p is adjacent to q, $a_{pq} > 0$; otherwise $a_{pq} = 0$. If there is a node p such that there exists a directed from it to any other node, $\mathscr{G}$ is said to contain a directed spanning-tree. The Laplacian matrix $\mathscr{L}$ defined as $\mathscr{L} = [l_{pq}] \in \mathscr{R}^{(\mathcal{M}+1)\times(\mathcal{M}+1)}$ with $l_{pq} = -a_{pq}, p \neq q$; and $l_{pq} = \sum_{q=1}^{\mathcal{M}+1} a_{pq}, \forall p = 1, \ldots, \mathcal{M}+1$.

B. Model Formulation and Basic Lemmas

We give some definitions of fractional calculus and lemmas that will be required later. Then, consensus issue of FOMASs is formulated via T-S fuzzy.

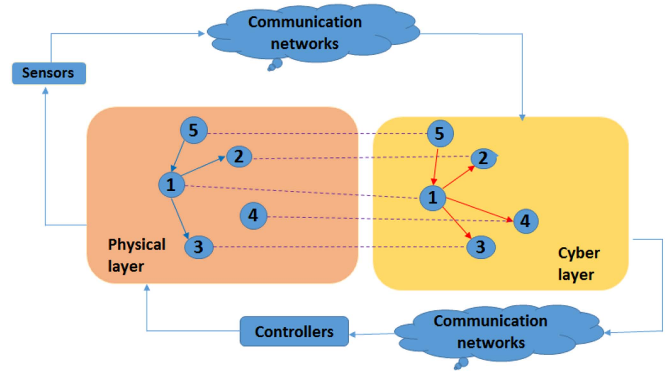

Fig. 1. Framework for networked agent systems with physical and cyber layers.

Definition 1 [11]: For $0 < \alpha \leq 1$, the Caputo fractional derivative is known as

$$ {}_{t_0}^{C}\mathcal{D}_t^{\alpha} h(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{h'(\xi)}{(t-\xi)^{\alpha}} d\xi, \qquad (1)$$

where $\Gamma(1-\alpha) = \int_0^{\infty} t^{-\alpha} e^{-\xi} d\xi$.

Definition 2 [14]: The Mittag-Leffler function are

$$\mathbb{E}_{\alpha}(z) = \sum_{m=0}^{\infty} \frac{z^m}{\Gamma(m\alpha+1)}, \qquad (2)$$

where $\alpha > 0$, and $z \in \mathcal{C}$.

Lemma 1 [14]: Let $\mathcal{V}(t)$ be a continuous function on $[t_0, +\infty)$ and satisfies ${}_{t_0}^{\mathcal{C}}D_t^{\alpha}\mathcal{V}(t) \leq \Theta\mathcal{V}(t)$, then

$$\mathcal{V}(t) \leq \mathcal{V}(t_0)\mathbb{E}_{\alpha}(\Theta(t-t_0))^{\alpha},$$

where $\alpha \in (0,1)$ and Θ constant.

Lemma 2 [12]: For $0 < \alpha < 1, t \in \mathscr{R}, t > 0$, we have

$$\lim_{t\to+\infty} \mathbb{E}_{\alpha}(t) \leq \lim_{t\to+\infty} \frac{1}{\alpha} e^{t^{\frac{1}{\alpha}}}.$$

Lemma 3 [13]: Let $x(t)$ be a continuous and derivable vector valued function. Then for any $t \geq t_0$, $\frac{1}{2}({}_{t_0}^{\mathcal{C}}D_t^{\alpha} x^T(t)x(t)) \leq x^T(t) {}_{t_0}^{\mathcal{C}}D_t^{\alpha} x(t)$, where $0 < \alpha < 1$.

The security control for T-S fuzzy FOMASs, which consists of cyber-attacks, is displayed in Fig. 1 with cyber layer $p = 1, 2, \ldots, \mathcal{M}, \mathcal{M}+1$. A network channel, controller and sensor are included in this framework. In fact, when an attack happens, the network can not operate properly, and then after a period of time, the networks attempts to restore or recovery process must reconstruct the network so that the network will work effectively. Several well-studied results on recovery mechanisms have been published. In recent years, there has been a significant increase in research on the security control of complex cyber-physical networks (see in [28]-[30], [44], [45]). It is noteworthy that several authors have recently investigated the security control problem for MASs in integer-order case (see in [26], [27], [31]–[34]), but there is no results for fractional case. Moreover, fractional-order case has better characteristics than corresponding integer-order case.

The fractional-order physical-plant model of $\mathcal{M}+1$ agents and $p(1 \leq p \leq \mathcal{M})$ followers as:

$$\begin{cases} {}_{t_0}^{\mathcal{C}}D_t^{\alpha}\Im_p(t) = \mathcal{A}\Im_p(t) + \mathcal{B}\tilde{u}_p(t), \\ \varphi_p(t) = \mathcal{C}\Im_p(t), \end{cases} \tag{3}$$

where $0 < \alpha < 1$, $\Im_p(t) \in \mathscr{R}^n$, $\varphi_p \in \mathscr{R}^m$, and $\tilde{u}_p(t) \in \mathscr{R}^r$ denotes, respectively, the state, output, and control inputs. $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ is the constant matrices. The control aims to establish distributed consensus tracking protocols $\tilde{u}_p(t), p = 1, \ldots, \mathcal{M}$ to make it asymptotic for the states of the followers to obey $\mathcal{M}+1$, which will satisfy the leaders [7],

$${}_{t_0}^{\mathcal{C}}D_t^{\alpha}\Im_{\mathcal{M}+1}(t) = \mathcal{A}\Im_{\mathcal{M}+1}(t). \tag{4}$$

C. T-S Fuzzy and T-S Fuzzy Control Models

Fuzzy logic systems directly address the imprecisions of the variables of input and output by describing them in linguistic terms with the fuzzy numbers (and fuzzy sets). The fractional physical plant system (3) is described in the T-S fuzzy approach are given:

Rule θ :

IF ϕ_1 is $\mathscr{M}_{\theta 1}$ and ϕ_2 is $\mathscr{M}_{\theta 2}$ and... and ϕ_k is $\mathscr{M}_{\theta k}$.

THEN

$$\begin{cases} {}_{t_0}^{\mathcal{C}}D_t^{\alpha}\Im_p(t) = \mathcal{A}_\theta\Im_p(t) + \mathcal{B}_\theta\tilde{u}_p(t), \\ \varphi_p(t) = \mathcal{C}_\theta\Im_p(t), \end{cases} \tag{5}$$

where $\phi_p(t)$ is the premise variable; $\mathscr{M}_{\theta k}$ for $\theta = 1, \ldots, \beta$ represents the fuzzy sets, β are IF-THEN laws; The constant matrices are $\mathcal{A}_\theta$, $\mathcal{B}_\theta$ and $\mathcal{C}_\theta$.

By the T-S fuzzy with final output processes, we have

$$\begin{cases} {}_{t_0}^{\mathcal{C}}D_t^{\alpha}\Im_p(t) = \sum_{\theta=1}^{\beta}\Psi_\theta(\phi_p(t))\left(\frac{\mathcal{A}_\theta\Im_p(t)+\mathcal{B}_\theta\tilde{u}_p(t)}{\sum_{\theta=1}^{\beta}\Psi_\theta(\phi_p(t))}\right), \\ \varphi_p(t) = \sum_{\theta=1}^{\beta}\Psi_\theta(\phi_p(t))\left(\frac{\mathcal{C}_\theta\Im_p(t)}{\sum_{\theta=1}^{\beta}\Psi_\theta(\phi_p(t))}\right), \end{cases} \tag{6}$$

where $\Psi_\theta(\phi_p(t)) = \prod_{\theta=1}^{m}\mathscr{M}_{\theta k}(\phi_p(t))$ with $\mathscr{M}_{\theta k}(\phi_p(t))$ representing the grade of memberships of ϕ_p in $\mathscr{M}_{\theta k}$, satisfy the following conditions:

$$\begin{cases} \sum_{\theta=1}^{\beta}\Psi_\theta(\phi_p(t)) > 0, \\ \Psi_\theta(\phi_p(t)) \geq 0, (\theta = 1, \ldots, \beta). \end{cases} \tag{7}$$

Let $\mu_\theta(\phi_p(t)) = \frac{\Psi_\theta(\phi_p(t))}{\sum_{\theta=1}^{\beta}\Psi_\theta(\phi_p(t))}$, then the expression (7) is written as

$$\begin{cases} {}_{t_0}^{\mathcal{C}}D_t^{\alpha}\Im_p(t) = \sum_{\theta=1}^{\beta}\mu_\theta(\phi_p(t))\big(\mathcal{A}_\theta\Im_p(t) + \mathcal{B}_\theta\tilde{u}_p(t)\big), \\ \varphi_p(t) = \sum_{\theta}^{\beta}\mu_\theta(\phi_p(t))\mathcal{C}_\theta\Im_p(t), \\ {}_{t_0}^{C}D_t^{\alpha}\Im_{\mathcal{M}+1} = \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))\mathcal{A}_\theta\Im_{\mathcal{M}+1}, \end{cases} \tag{8}$$

where

$$\begin{cases} \sum_{\theta=1}^{\beta}\mu_\theta(\phi_p(t)) = 1, \\ \mu_\theta(\phi_p(t)) \geq 0, (\theta = 1, \ldots, \beta), \end{cases}$$

where $\mu_\theta(\phi_p(t))$ are IF-THEN rules weights.

The fuzzy control design of distributed consensus tracking protocals $\tilde{u}_p(t)$ is given as

Rule θ :

IF ϕ_1 is $\mathscr{M}_{\theta 1}$ and ϕ_2 is $\mathscr{M}_{\theta 2}$ and... and ϕ_k is $\mathscr{M}_{\theta k}$.

THEN

$$\tilde{u}_p(t) = \xi\sum_{q=1}^{\mathcal{M}+1}\mathcal{K}_\theta a_{pq}^{(\hat{\sigma}(k))}(\Im_q(t) - \Im_p(t)), \tag{9}$$

where $\mathcal{K}_\theta$ are the control gain matrices, ξ are coupling strengths, $a_{pq}^{(\hat{\sigma}(k))}$ is the adjacent matrix representing the communication network via attacks.

The final output can be described as the fuzzy prediction controller by,

$$\tilde{u}_p(t) = \xi\sum_{\theta=1}^{\beta}\mu_\theta(\phi_p(t))\sum_{q=1}^{\mathcal{M}+1}\mathcal{K}_\theta a_{pq}^{(\hat{\sigma}(k))}(\Im_q(t) - \Im_p(t)). \tag{10}$$

Substitute (10) in (8), we obtain the complete controlled T-S fuzzy system as:

$$\begin{cases} {}_{t_0}^{\mathcal{C}}D_t^{\alpha}\Im_p(t) &= \sum_{\theta=1}^{\beta}\mu_\theta(\phi_p(t))(\mathcal{A}_\theta\Im_p(t) \\ &\quad +\xi\mathcal{B}_\theta\sum_{q=1}^{\mathcal{M}+1}\mathcal{K}_\theta a_{pq}^{(\hat{\sigma}(k))}(\Im_q(t) - \Im_p(t))) \\ \varphi_p(t) &= \sum_{\theta=1}^{\beta}\mu_\theta(\phi_p(t))\mathcal{C}_\theta\Im_p(t). \end{cases} \tag{11}$$

The following distributed state T-S fuzzy observer is developed for the followers p $(1 \leq p \leq \mathcal{M})$ to estimate the unknown system states $\Im_p(t)$ in system (3):

$$\begin{cases} {}_{t_0}^{\mathcal{C}}D_t^{\alpha}\widehat{\Im}_p(t) &= \sum_{\theta=1}^{\beta}\mu_\theta(\phi_p(t))(\mathcal{A}_\theta\widehat{\Im}_p(t) \\ &\quad +\xi\mathcal{B}_\theta\sum_{q=1}^{\mathcal{M}+1}\mathcal{K}_\theta a_{pq}^{(\hat{\sigma}(k))}(\widehat{\Im}_q(t) - \widehat{\Im}_p(t))) \\ &\quad +\tilde{\xi}\Omega_\theta\sum_{q=1}^{\mathcal{M}+1}\mathcal{K}_\theta a_{pq}^{(\check{\sigma}(k))}(\rho_q(t) - \rho_p(t)) \\ \hat{\varphi}_p(t) &= \sum_{\theta=1}^{\beta}\mu_\theta(\phi_p(t))\mathcal{C}_\theta\widehat{\Im}_p(t), \end{cases} \tag{12}$$

where $\widehat{\Im}_p(t)$ is the observers state for agent p, and $\rho_p(t) = \hat{\varphi}_p(t) - \varphi_p(t) = \mathcal{C}(\widehat{\Im}_p(t) - \Im_p(t))$,$\Omega_\theta$ are the observer control gain matrices, $\tilde{\xi}$ is coupling strength, $a_{pq}^{(\check{\sigma}(k))}$ is the adjacent matrix representing the observer-based communication network via attacks.

Remark 1: The states of system dynamics are not always completely accessible in a realistic application. As an outcome, the observer-based control technique has gradually evolved into a valuable tool for networked control systems. A high-order system can be viewed as FOMAS (3) and high dimensional system states can be seen in (12). It is well known that in practice the systems state $\Im_p(t)$ can be difficult to obtain, therefore the state observer $\widehat{\Im}_p(t)$ is intended in this paper to estimate the state $\Im_p(t)$ (kindly refer [7]). In addition, the observer (12) also discusses gains $\mathcal{K}_\theta$ and Ω_θ, that are able to effectively improve the control characteristics for FOMAS and decrease the conservativeness of the output feedback control design. In this study, we focused on the general dynamics of the leader-follower MAS, which varies from the established observer-based

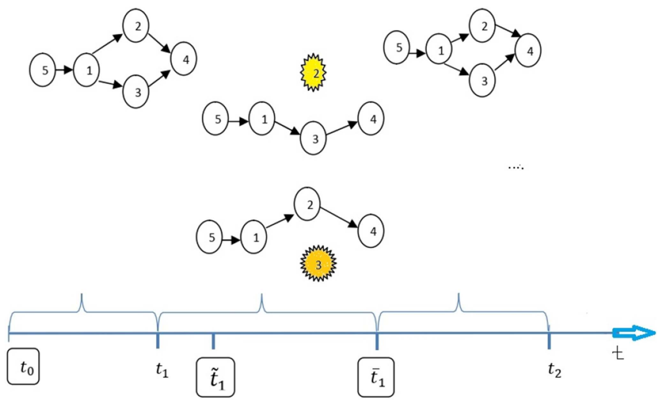

Fig. 2. Attack to communication network of controllers and observers.

model in previous studies ([24], [25]). Compared with the previous studies in ([24], [25]), our obtained criteria improve the previous results. In this paper the malicious cyber-attacks are also considered for realizing output feedback controller.

However, the attacks method of targeting the controller is safer since manipulating the control signal $u(t)$ expressly reveals the system to vulnerabilities. Fig. 2 depicts the communication networks of controllers and observers during p^{th} attacks in the time interval $[\tilde{t}_p, \bar{t}_p]$. The enhanced network containing of 4 followers and one leader in the presence of attacks on nodes. Node 2 (shade star) is attacked to the communication channel of controllers and node 3 (shade star) is attacked to the communication channel of observers. Here, t_0 and t_p are respectively initial, and p^{th} malicious attack occurs. $\tilde{t}_p$ and $\bar{t}_p$, $p = 1, 2, \ldots$, are respectively instants of time during which the p^{th} attack and the node functions are recovered. It is believed that malicious assaults will have an independent impact on controller and observation channels. Attacks can be seen to occur at time instant t_1, but the cyber command centre notices them at $\bar{t}_1$. Then, beginning with $\bar{t}_1$, the repair system will be turned on. From $\bar{t}_1$ to $\tilde{t}_1$, the communication graphs are discontinuous. Specifically, node 3 is destroyed in the observation communication network and node 2 becomes inactive in the communication network of control inputs. The effect of the attacks will be eliminated at $\tilde{t}_1$, and during the time interval $[\tilde{t}_1, t_2]$, the topology of the whole network will be recovered back to its initial setting, until the next attacks happen at t_2.

By using Kronecker product, from (11) and (12) are given:

$$
\begin{aligned}
{}_{t_0}^{\mathcal{C}}D_t^{\alpha}\Im(t) = \sum_{\theta=1}^{\beta}\mu_\theta(\phi_p(t))((\mathcal{I}_\mathcal{M}\otimes\mathcal{A}_\theta)\Im(t) \\
- \xi(\mathscr{L}_{\hat{\sigma}(k)}\otimes\mathcal{B}_\theta\mathcal{K}_\theta)\widetilde{\Im}(t)),
\end{aligned} \tag{13}
$$

and

$$
\begin{aligned}
{}_{t_0}^{\mathcal{C}}D_t^{\alpha}\widehat{\Im}(t) = \sum_{\theta=1}^{\beta}\mu_\theta(\phi_p(t))((\mathcal{I}_\mathcal{M}\otimes\mathcal{A}_\theta)\widehat{\Im}(t) \\
- \xi(\mathscr{L}_{\hat{\sigma}(k)}\otimes\mathcal{B}_\theta\mathcal{K}_\theta)\tilde{x}(t) - \tilde{\xi}(\mathscr{L}_{\check{\sigma}(k)}\otimes\Omega_\theta)\rho(t)),
\end{aligned} \tag{14}
$$

where $\Im(t) = (\Im_1^T(t), \ldots, \Im_\mathcal{M}^T(t))^T$, $\widehat{\Im}(t) = (\widehat{\Im}_1^T(t), \ldots, \widehat{\Im}_\mathcal{M}^T(t))^T$, $\rho(t) = (\rho_1^T(t), \ldots, \rho_\mathcal{M}^T(t))^T$ and $\widetilde{\Im}(t) = (\widehat{\Im}_\mathcal{M}^T(t), \Im_{\mathcal{M}+1}^T(t))^T$, $\hat{\mathscr{L}}_{\hat{\sigma}(k)} = [\begin{smallmatrix}\bar{\mathscr{L}}_{\hat{\sigma}(k)} & \chi \\ 0_\mathcal{M}^T & 0\end{smallmatrix}]$, $\hat{\mathscr{L}}_{\check{\sigma}(k)} = [\begin{smallmatrix}\bar{\mathscr{L}}_{\check{\sigma}(k)} & \chi \\ 0_\mathcal{M}^T & 0\end{smallmatrix}]$, $\chi = (\chi_1, \ldots, \chi_\mathcal{M})^T$, in which $\chi_p = 1$ if a relation from the leader to the follower p exists; otherwise, $\chi_p = 0$.

Define $\omega_p(t) = \Im_p(t) - \Im_{\mathcal{M}+1}(t)$, $\varpi_p(t) = \Im_p(t) - \widehat{\Im}_p(t)$, $\omega^T(t) = (\omega_1^T(t), \ldots, \omega_\mathcal{M}^T(t))^T$, $\varpi^T(t) = (\varpi_1^T(t), \ldots, \varpi_\mathcal{M}^T(t))^T$.

From (13) and (14) can be written as:

$$
\begin{aligned}
{}_{t_0}^{\mathcal{C}}D_t^{\alpha}\varpi(t) = \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))((\mathcal{I}_\mathcal{M}\otimes\mathcal{A}_\theta)\varpi(t) \\
- \tilde{\xi}(\mathscr{L}_{\check{\sigma}(k)}\otimes\Omega_\theta\mathcal{C}_\theta)\varpi(t)),
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
{}_{t_0}^{\mathcal{C}}D_t^{\alpha}\omega(t) = \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))((\mathcal{I}_\mathcal{M}\otimes\mathcal{A}_\theta)\omega(t) \\
- \xi(\mathscr{L}_{\hat{\sigma}(k)}\otimes\mathcal{B}_\theta\mathcal{K}_\theta)\tilde{x}(t)),
\end{aligned} \tag{16}
$$

with

$$
\begin{aligned}
&(\mathscr{L}_{\hat{\sigma}(k)}\otimes\mathcal{B}_\theta\mathcal{K}_\theta)\tilde{x}(t) \\
&= (\mathscr{L}_{\hat{\sigma}(k)}\otimes\mathcal{B}_\theta\mathcal{K}_\theta)(\tilde{x}(t) - 1_{\mathcal{M}+1}\otimes x_{\mathcal{M}+1}(t)) \\
&= (\mathscr{L}_{\hat{\sigma}(k)}\otimes\mathcal{B}_\theta\mathcal{K}_\theta)(\hat{x}(t) - 1_\mathcal{M}\otimes x_{\mathcal{M}+1}) \\
&= (\mathscr{L}_{\hat{\sigma}(k)}\otimes\mathcal{B}_\theta\mathcal{K}_\theta)(\omega(t) - \varpi(t)).
\end{aligned} \tag{17}
$$

From (15)-(17) we have,

$$
{}_{t_0}^{\mathcal{C}}D_t^{\alpha}\hat{\omega}(t) = \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))\mathscr{A}\hat{\omega}(t),
$$

where $\mathscr{A} = [\begin{smallmatrix}\Im_1 & 0_{n\mathcal{M}\times n\mathcal{M}} \\ \xi(\mathscr{L}_{\hat{\sigma}(k)}\otimes\mathcal{B}_\theta\mathcal{K}_\theta) & \Im_2\end{smallmatrix}]$, $\hat{\omega}(t) = [\varpi^T(t), \omega^T(t)]^T$, $\Im_1(\mathcal{I}_\mathcal{M}\otimes\mathcal{A}_\theta) - \tilde{\xi}(\mathscr{L}_{\check{\sigma}(k)}\otimes\Omega_\theta\mathcal{C}_\theta)$, $(\mathcal{I}_\mathcal{M}\otimes\mathcal{A}_\theta) - \xi(\mathscr{L}_{\hat{\sigma}(k)}\otimes\mathcal{B}_\theta\mathcal{K}_\theta)$.

III. Main Result

Theorem 1: For positive scalars $\vartheta, \hat{\vartheta}, \eta, \bar{\eta}, \tilde{\eta}, \hat{\eta}, \gamma, \alpha$, the positive-definite matrices W, U with $\mathcal{K}_\theta = \mathcal{B}_\theta^T U^{-1}$ and $\Omega_\theta = W^{-1}\mathcal{C}_\theta^T$, FOMAS (3) and (4) can be achieved the consensus using tracking control (9), if the following inequalities hold,

$$
W\mathcal{A}_\theta + \mathcal{A}_\theta^T W - \vartheta\mathcal{C}_\theta^T\mathcal{C}_\theta + \eta W < 0, \tag{18}
$$

$$
\mathcal{A}_\theta U + U\mathcal{A}_\theta^T - \hat{\vartheta}\mathcal{B}_\theta\mathcal{B}_\theta^T + \bar{\eta}U < 0, \tag{19}
$$

$$
\varXi = \begin{bmatrix} -\tilde{\eta}(\Upsilon\otimes W) & * \\ \Lambda & -\tau\hat{\eta}(\Upsilon\otimes U^{-1}) \end{bmatrix} < 0, \tag{20}
$$

where $\Lambda = \tau(\xi\Upsilon\mathscr{L}\otimes U^{-1}\mathcal{B}_\theta\mathcal{B}_\theta^T U^{-1})$ and for each $k \in \mathscr{N}$,

$$
(\eta_{\min}(t_k - \bar{t}_{k-1}))^{\frac{1}{\alpha}} - (\Theta_{\min}(\bar{t}_k - t_k))^{\frac{1}{\alpha}} - \epsilon < 0, \tag{21}
$$

and $\Theta_{\min} = \min\{\Theta_1, \Theta_2\}$.

The following optimization problem is obtained by solving:

Minimize Θ_1, subject to

$$W\mathcal{A}_\theta + \mathcal{A}_\theta^T W - \xi\hat{\psi}\mathcal{C}_\theta^T\mathcal{C}_\theta - \Theta_1 W < 0. \tag{22}$$

Minimize Θ_2, subject to

$$\mathcal{A}_\theta^T U^{-1} + U^{-1}\mathcal{A}_\theta - \hat{\xi}\check{\psi}U^{-1}\mathcal{B}_\theta\mathcal{B}_\theta^T U^{-1} - \Theta_2 U^{-1} < 0, \tag{23}$$

where $\hat{\psi}_{\min} = \frac{\hat{\lambda}_{\min}}{\gamma_{\min}}, \hat{\lambda}_{\min} = \min_{\hat{\sigma}(k)}\{\mathscr{L}_{\hat{\sigma}(k)}^T\Upsilon + \Upsilon\mathscr{L}_{\hat{\sigma}(k)}\}$, $\check{\psi}_{\min} = \frac{\check{\lambda}_{\min}}{\gamma_{\min}}, \check{\lambda}_{\min} = \min_{\check{\sigma}(k)}\{\mathscr{L}_{\check{\sigma}(k)}^T\Upsilon + \Upsilon\mathscr{L}_{\check{\sigma}(k)}\}, \gamma_{\min} = \min_{1\le p\le\mathcal{M}}\{\gamma_p\}$.

Proof. Consider a Lyapunov function:

$$V(t) = \varpi^T(t)(\Upsilon\otimes W)\varpi(t) + \tau\omega^T(t)(\Upsilon\otimes U^{-1})\omega(t), \tag{24}$$

where U and W are positive definite. Here denote $V_\pi = \varpi^T(t)(\Upsilon\otimes W)\varpi(t)$, and $V_\delta = \omega^T(t)(\Upsilon\otimes U^{-1})\omega(t)$. For $t\in[\bar{t}_{k-1}, t_k), k\in\mathcal{N}$ without attacks occur $\mathscr{L}_{\hat{\sigma}(k)} = \mathscr{L}_{\check{\sigma}(k)} = \mathscr{L}$.

By applying Lemma 3, one has

$$\begin{aligned}
{}_{t_0}^{C}D_t^\alpha V_\pi(t) &= {}_{t_0}^{C}D_t^\alpha(\varpi^T(t)(\Upsilon\otimes W)\varpi(t))\\
&\le 2(\varpi^T(t)(\Upsilon\otimes W){}_{t_0}^{\mathcal{C}}D_t^\alpha\varpi(t))\\
&\le 2\sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))(\varpi^T(t)(\Upsilon\otimes W)((\mathcal{I}_\mathcal{M}\otimes\mathcal{A}_\theta)\\
&\quad - \hat{\xi}(\mathscr{L}\otimes\Omega_\theta\mathcal{C}_\theta))\varpi(t))\\
&\le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))\varpi^T(t)(\Upsilon\otimes(W\mathcal{A}_\theta + \mathcal{A}_\theta^T W)\\
&\quad - 2\hat{\xi}(\Upsilon\mathscr{L}\otimes W\Omega_\theta\mathcal{C}_\theta))\varpi(t)\\
&\le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))\varpi^T(t)(\Upsilon\otimes(W\mathcal{A}_\theta + \mathcal{A}_\theta^T W)\\
&\quad - 2(\Upsilon\mathscr{L}\otimes\mathcal{C}_\theta^T\mathcal{C}_\theta))\varpi(t)\\
&\le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))\varpi^T(t)(\Upsilon\otimes(W\mathcal{A}_\theta + \mathcal{A}_\theta^T W\\
&\quad - \frac{\hat{\xi}}{\gamma_{\max}}\lambda_{\min}(\Upsilon\mathscr{L} + \mathscr{L}^T\Upsilon)\Upsilon\mathcal{C}_\theta^T\mathcal{C}_\theta))\varpi(t).
\end{aligned} \tag{25}$$

For $\hat{\xi} > \vartheta/\zeta, \zeta = \frac{\lambda_{\min}(\Upsilon\mathscr{L}+\mathscr{L}^T\Upsilon)}{\gamma_{\max}}$, where $\gamma_{\max} = \max_{1\le p\le\mathcal{M}}\{\gamma_p\}, \Upsilon = diag\{\gamma_1, \gamma_2, \ldots, \gamma_\mathcal{M}\}, \mathcal{L}^T\gamma = 1_\mathcal{M}$.

It follows that

$$\begin{aligned}
{}_{t_0}^{\mathcal{C}}D_t^\alpha V_\pi(t) \le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))\varpi^T(t)(\Upsilon\otimes(W\mathcal{A}_\theta + \mathcal{A}_\theta^T W\\
- \vartheta\mathcal{C}_\theta^T\mathcal{C}_\theta))\varpi(t).
\end{aligned} \tag{26}$$

Then,

$${}_{t_0}^{\mathcal{C}}D_t^\alpha V_\pi(t) \le -\sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))(\eta+\tilde{\eta})\varpi^T(t)(\Upsilon\otimes W)\varpi(t), \tag{27}$$

where $\tilde{\eta}$ is a constant, $0 < \tilde{\eta} \ll \eta$.

Next, taking the fractional derivative of $\mathcal{V}_\delta$ with system (16), one has

$$\begin{aligned}
{}_{t_0}^{\mathcal{C}}D_t^\alpha V_\delta(t) &= {}_{t_0}^{\mathcal{C}}D_t^\alpha\big(\omega^T(t)(\Upsilon\otimes U^{-1})\omega(t)\big)\\
&\le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))2\xi\big(\omega^T(t)(\Upsilon\mathscr{L}\otimes U^{-1}\mathcal{B}_\theta\mathcal{K}_\theta\big)\varpi(t)\\
&\quad + \omega^T(t)\big(\Upsilon\mathscr{L}\otimes U^{-1}\mathcal{B}_\theta\mathcal{K}_\theta\big)\omega(t))\\
&\le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))\big(2\xi\omega^T(t)(\Upsilon\mathscr{L}\otimes U^{-1}\mathcal{B}_\theta\mathcal{B}_\theta^T U^{-1}\big)\\
&\quad \times\varpi(t) + \omega^T(t)(\mathcal{A}_\theta U^{-1} + U^{-1}\mathcal{A}_\theta)\\
&\quad - \frac{\xi}{\gamma_{\max}}\lambda_{\min}(\Upsilon\mathscr{L} + \mathscr{L}^T\Upsilon)\\
&\quad \times\Upsilon U^{-1}\mathcal{B}_\theta\mathcal{B}_\theta^T U^{-1})\omega(t)).
\end{aligned} \tag{28}$$

For $\xi > \hat{\vartheta}/\hat{\zeta}, \hat{\zeta} = \frac{\lambda_{\min}(\Upsilon\mathscr{L}+\mathscr{L}\Upsilon)}{\gamma_{\max}}$, it follows that

$$\begin{aligned}
{}_{t_0}^{\mathcal{C}}D_t^\alpha V_\delta(t) \le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))\big(2\eta\omega^T(t)(\Upsilon\mathscr{L}\otimes U^{-1}\mathcal{B}_\theta\mathcal{B}_\theta^T U^{-1}\big)\\
\times\varpi(t) - (\bar{\eta}+\hat{\eta})(\Upsilon\otimes U^{-1})\omega(t)),
\end{aligned} \tag{29}$$

where $\hat{\eta}$ is a constant, $0 < \hat{\eta} \ll \bar{\eta}$.

From (27) and (29), we obtain

$$\begin{aligned}
{}_{t_0}^{\mathcal{C}}D_t^\alpha V(t) &\le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))(-(\eta+\tilde{\eta})\varpi^T(t)(\Upsilon\otimes W)\varpi(t)\\
&\quad + 2\eta\omega^T(t)\big(\Upsilon\mathscr{L}\otimes U^{-1}\mathcal{B}_\theta\mathcal{B}_\theta^T U^{-1}\big)\varpi(t)\\
&\quad - (\bar{\eta}+\hat{\eta})(\Upsilon\otimes U^{-1})\omega(t))\\
&\le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))(-\eta\varpi^T(t)(\Upsilon\otimes W)\varpi(t) + \tilde{\omega}^T\Xi\tilde{\omega}(t)\\
&\quad - \bar{\eta}\varpi^T(t)(\Upsilon\otimes U^{-1}))\\
&< -\sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))(\eta\varpi^T(t)(\Upsilon\otimes W)\varpi(t)\\
&\quad + \bar{\eta}\varpi^T(t)(\Upsilon\otimes U^{-1}))\\
&< -\sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))\eta_{\min}V(t)
\end{aligned}$$

$${}^{\mathcal{C}}D_t^\alpha V(t) < -\eta_{\min}V(t), \quad t\in[\bar{t}_{k-1}, t_k), \tag{30}$$

where $\eta_{\min} = \min\{\eta, \bar{\eta}\}$. By applying Lemma 1, we get

$$V(t_k) \le V(\bar{t}_{k-1})\mathbb{E}_\alpha(-\eta_{min}(t_k - \bar{t}_{k-1}))^\alpha. \tag{31}$$

According to Lemma 2, we get

$$V(t_k) \le V(\tilde{t}_{k-1})\frac{1}{\alpha}e^{-(\eta_{\min}(t_k-\bar{t}_{k-1}))^{\frac{1}{\alpha}}}. \tag{32}$$

When $t\in[t_k, \bar{t}_k)$, both controllers and observers communication network is destroyed by malicious attacks, i.e. $\mathscr{L}_{\hat{\sigma}(k)}$ and $\mathscr{L}_{\check{\sigma}(k)}$ should both be considered. Calculating the Caputo fractional derivative of V_φ and V_δ along the trajectory of system (15) and (16) by selecting Θ_1 and Θ_2 in (22) and (23) respectively, we get

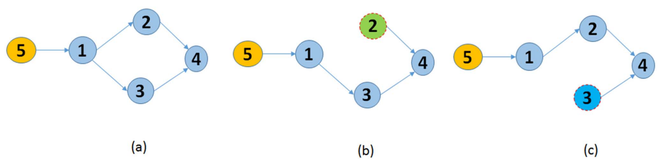

Fig. 3. The communication topology.

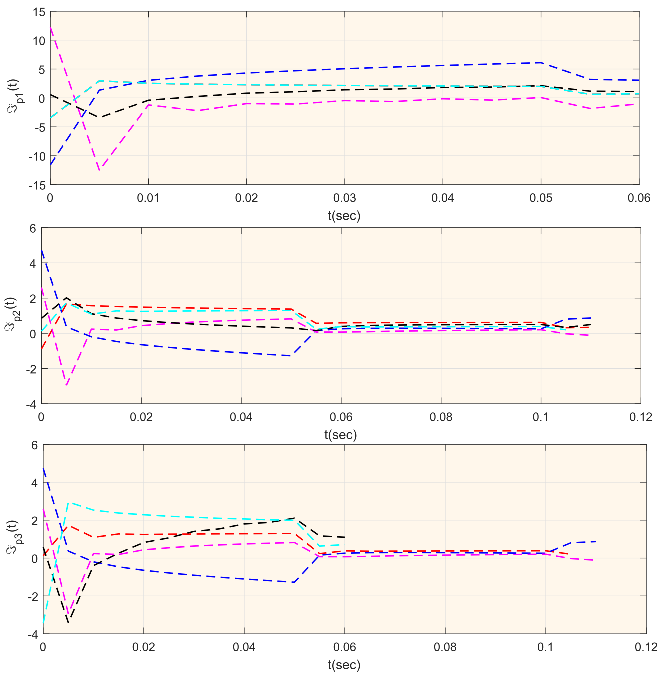

Fig. 4. State trajectories for FOMASs when attacks occur with no security control, where $\Im_p(t) = (\Im_{p1}(t), \Im_{p2}(t), \Im_{p3}(t))^T, p = 1, 2, \ldots, 5$.

$$\begin{aligned}
{}_{t_0}^{C}D_t^{\alpha}V_{\pi}(t) &= {}_{t_0}^{C}D_t^{\alpha}(\varpi^T(t)(\Upsilon\otimes W)\varpi(t))\\
&\le 2(\varpi^T(t)(\Upsilon\otimes W){}_{t_0}^{C}D_t^{\alpha}\varpi(t))\\
&\le 2\sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))(\varpi^T(t)(\Upsilon\otimes W)((\mathcal{I}_{\mathcal{M}}\otimes\mathcal{A}_\theta)\\
&\quad - \hat{\xi}(\mathscr{L}_{\breve{\sigma}(k)}\otimes\Omega_\theta\mathcal{C}_\theta))\varpi(t))\\
&\le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))(\Theta_1-\tilde{\eta})\varpi^T(t)(\Upsilon\otimes W)\varpi(t), \quad (33)
\end{aligned}$$

and

$$\begin{aligned}
{}_{t_0}^{C}D_t^{\alpha}V_{\delta}(t) &= {}_{t_0}^{C}D_t^{\alpha}\big(\omega^T(t)(\Upsilon\otimes U^{-1})\omega(t)\big)\\
&\le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))2\xi\big(\omega^T(t)(\Upsilon\mathscr{L}_{\hat{\sigma}(k)}\otimes U^{-1}\mathcal{B}_\theta\mathcal{K}_\theta)\varpi(t)\\
&\quad + \omega^T(t)\big(\Upsilon\mathscr{L}_{\hat{\sigma}(k)}\otimes U^{-1}\mathcal{B}_\theta\mathcal{K}_\theta\big)\omega(t)\big)\\
&\le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))\big(2\xi\omega^T(t)(\Upsilon\mathscr{L}_{\hat{\sigma}(k)}\otimes U^{-1}\mathcal{B}_\theta\mathcal{B}_\theta^T U^{-1})\\
&\quad \times\varpi(t) + (\Theta_2-\hat{\eta})\omega^T(t)(\Upsilon\otimes U^{-1})\omega(t)\big). \quad (34)
\end{aligned}$$

From (33) and (34), one has

$$\begin{aligned}
{}_{t_0}^{C}D_t^{\alpha}V(t) &\le \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))(\Theta_1\varpi^T(t)(\Upsilon\otimes W)\varpi(t)\\
&\quad + \widetilde{\omega}^T\Xi_{\sigma(k)}\widetilde{\omega}(t) + \tau\Theta_2\omega^T(t)(\Upsilon\otimes U^{-1})\omega(t))\\
&< \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))(\Theta_1\varpi^T(t)(\Upsilon\otimes W)\varpi(t)\\
&\quad + \tau\Theta_2\varpi^T(t)(\Upsilon\otimes U^{-1})\omega(t))\\
&< \sum_{\theta=1}^{\beta}\mu_\theta(\phi(t))\Theta_{\min}V(t)\\
&< \Theta_{\min}V(t). \quad (35)
\end{aligned}$$

Applying Lemma 1 in (35) we get

$$V(\bar{t}_k) \le V(t_k)\mathbb{E}_\alpha(\Theta_{min}(\bar{t}_k - t_k))^{\alpha}. \quad (36)$$

By applying Lemma 2, one has

$$\begin{aligned}
V(\bar{t}_k) &\le V(t_k)\frac{1}{\alpha}e^{(\Theta_{\min}(\bar{t}_k-t_k))^{\frac{1}{\alpha}}}\\
&\le V(\bar{t}_{k-1})\frac{1}{\alpha}e^{-(\eta_{\min}(t_k-\bar{t}_{k-1}))^{\frac{1}{\alpha}}}\frac{1}{\alpha}e^{(\Theta_{\min}(\bar{t}_k-t_k))^{\frac{1}{\alpha}}}\\
&\le V(\bar{t}_{k-1})\frac{1}{\alpha^2}e^{-\left((\eta_{\min}(t_k-\bar{t}_{k-1}))^{\frac{1}{\alpha}}-(\Theta_{\min}(\bar{t}_k-t_k))^{\frac{1}{\alpha}}\right)}\\
&< \frac{1}{\alpha^2}e^{-\epsilon}V(\bar{t}_{k-1}) < \frac{1}{\alpha^2}e^{-k\epsilon}V(t_0).
\end{aligned}$$

Therefore, the FOMAS (3) and (4) can be reached the consensus, i.e., $\omega(t)$ and $\varpi(t)$ are converge to zero.

Corollary 1: For positive scalars $\vartheta, \hat{\vartheta}$, $\eta, \bar{\eta}, \tilde{\eta}, \hat{\eta}, \tau$ with $\mathcal{K}_\theta = \mathcal{B}_\theta^T U^{-1}$, $\Omega_\theta = W^{-1}\mathcal{C}_\theta^T$, and attacks occur only in the communication channel for the controllers, FOMAS (3) and (4) can achieve the consensus using tracking control (9), if (18)-(20) holds in Theorem 1 and for each $k \in \mathscr{N}$, $(\eta_{\min}(t_k - \bar{t}_{k-1}))^{\frac{1}{\alpha}} - (\Theta_1(\bar{t}_k - t_k))^{\frac{1}{\alpha}} - \epsilon < 0$. The following optimization problem is obtained by solving:

Minimize Θ_1, subject to

$$W\mathcal{A}_\theta + \mathcal{A}_\theta^T W - \xi\hat{\psi}\mathcal{C}_\theta^T\mathcal{C}_\theta - \Theta_1 W < 0. \quad (37)$$

where $\hat{\psi}_{\min} = \frac{\hat{\lambda}_{\min}}{\gamma_{\min}}$, $\hat{\lambda}_{\min} = \min_{\hat{\sigma}(k)}\{\mathscr{L}_{\hat{\sigma}(k)}^T\Upsilon + \Upsilon\mathscr{L}_{\hat{\sigma}(k)}\}$, $\gamma_{\min} = \min_{1\le p\le\mathcal{M}}\{\gamma_p\}$.

Corollary 2: For positive scalars $\vartheta, \hat{\vartheta}$, $\eta, \bar{\eta}, \tilde{\eta}, \hat{\eta}, \tau$ with $\mathcal{K}_\theta = \mathcal{B}_\theta^T U^{-1}$, $\Omega_\theta = W^{-1}\mathcal{C}_\theta^T$, and attacks occur only in the communication network for the observers, FOMAS (3) and (4) can achieve the consensus using tracking control (9), if (18)-(20) in Theorem 1 hold and for each $k \in \mathscr{N}$, $(\eta_{\min}(t_k - \bar{t}_{k-1}))^{\frac{1}{\alpha}} - (\Theta_2(\bar{t}_k - t_k))^{\frac{1}{\alpha}} - \epsilon < 0$. The following optimization problem is obtained by solving:

Minimize Θ_2, subject to

$$\mathcal{A}_\theta^T U^{-1} + U^{-1}\mathcal{A}_\theta - \hat{\xi}\check{\psi}U^{-1}\mathcal{B}_\theta\mathcal{B}_\theta^T U^{-1} - \Theta_2 U^{-1} < 0, \quad (38)$$

where $\check{\psi}_{\min} = \frac{\check{\lambda}_{\min}}{\gamma_{\min}}$, $\check{\lambda}_{\min} = \min_{\breve{\sigma}(k)}\{\mathscr{L}_{\breve{\sigma}(k)}^T\Upsilon + \Upsilon\mathscr{L}_{\breve{\sigma}(k)}\}$, $\gamma_{\min} = \min_{1\le p\le\mathcal{M}}\{\gamma_p\}$.

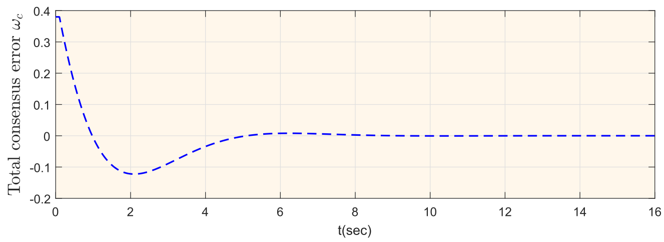

Fig. 5. Evolution of total consensus errors ω_c.

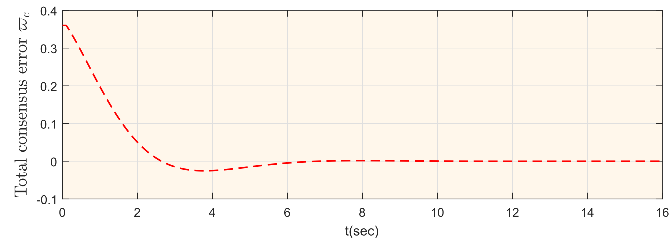

Fig. 6. Evolution of total observer errors ϖ_c.

*Remark 2:*In special case, when cyber attacks can not impact the security of topologies of the communication networks of both controllers and observers ([24], [25]) formation control of observer-based FOMAS without T-S fuzzy system. Next our investigated to Mittag-Leffler sense of T-S fuzzy observer-based FOMAS and also our proposed method valid with $\alpha \in (0,1]$ in ([24], [25]), so the following Corollary.

Corollary 3: For positive scalars $\vartheta, \hat{\vartheta}, \ \eta, \bar{\eta}, \tilde{\eta}, \hat{\eta}, \tau$ with $\mathcal{K}_\theta = \mathcal{B}_\theta^T U^{-1}$, and $\Omega_\theta = W^{-1}\mathcal{C}_\theta^T$, FOMAS (3) and (4) can be achieved the Mittag-Leffler sense of consensus using tracking control (9), if (18)-(20) holds and $\eta_{\min} > 0$.

Proof. Consider the Lyapunov functional (24) and taking the fractional derivative on system (15) and (16), we obtain that

$$ {}_{t_0}^{C}D_t^\alpha V(t) < -\eta_{\min} V(t), $$

where $\eta_{\min} = \min\{\eta, \bar{\eta}\}$. By using Lemma 1, we get

$$ V(t) \leq V(t_0)\mathbb{E}_\alpha(-\eta_{min}(t-t_0))^\alpha, \tag{39} $$

where $\eta_{\min} = \min\{\eta, \bar{\eta}\} > 0$. It follows from [14], the inequality (39) are satisfied. Therefore, the FOMAS (3) and (4) can be reached the consensus of Mittag-Leffler sense.

IV. Numerical Examples

Two numerical examples are used in this section to demonstrate the effectiveness of the theoretical results achieved.

Example 1: We consider FOMAS system (3) and (4) with $\Im_p = [\Im_{p1}, \Im_{p2}, \Im_{p3}]^T, p = 1, \ldots, 5$. Having 4 followers and one leader, allow these agents to get information from their neighbor in accordance with their communication topology. We analyze two scenarios, 1) When the cyber system is not under attack, the communication networks is represented in Fig. 3(a). 2) When a cyber system is under attack, the communication topology is depicted in Fig. 3(b) and (c). Suppose that attacks occur in the time interval $[t_p, \bar{t}_p]$, we take $p \in \mathscr{N}$, $t_p = 0.7(p-1)$ and $\bar{t}_p = 0.7(p-1) + 0.5$.

The T-S fuzzy model is:

Rule $\mathscr{R}^1$

IF $\Im_{p1}$ is $\mathscr{M}_1$, THEN $\begin{cases} {}^C D_t^\alpha \Im(t) = \mathcal{A}_1 \Im(t) + \mathcal{B}_1 \tilde{u}(t), \\ \varphi(t) = \mathcal{C}_1 \Im(t). \end{cases}$

Rule $\mathscr{R}^2$

IF $\Im_{p1}$ is $\mathscr{M}_2$, THEN $\begin{cases} {}^C D_t^\alpha \Im(t) = \mathcal{A}_2 \Im(t) + \mathcal{B}_2 \tilde{u}(t), \\ \varphi(t) = \mathcal{C}_2 \Im(t), \end{cases}$

where $\Im(t) = (\Im_{p1}, \Im_{p2}, \Im_{p3})$,

$$ \mathcal{A}_1 = \mathcal{A}_2 = \begin{bmatrix} -2.97 & 0.92 & -0.04 \\ -0.93 & 0 & -0.012 \\ 0.37 & -4.73 & -1.79 \end{bmatrix}, $$

$$ \mathcal{B}_1 = \mathcal{B}_2 = \begin{bmatrix} -0.05 & 0.90 & 1.59 \\ 0 & 0 & 0 \\ -1.80 & 1.70 & -2 \end{bmatrix}, $$

$$ \mathcal{C}_1 = \mathcal{C}_2 = \begin{bmatrix} 2.50 & 0.98 & 1.70 \\ 0.73 & 1.70 & 0.97 \end{bmatrix}, $$

$\mathscr{M}_1 = \frac{1}{2}(1 + \frac{\Im_{p1}}{d}), \mathscr{M}_2 = \frac{1}{2}(1 - \frac{\Im_{p1}}{d}), \quad \Im_{p1} \in [-d, d]$ with $d > 0$.

The distributed control is taken as:

Rule $\mathscr{R}^1$:

IF $\Im_{p1}$ is $\mathcal{M}_1$, THEN $\tilde{u}(t) = \xi \mathcal{K}_1 \Im(t)$,

Rule $\mathscr{R}^2$:

IF $\Im_{p1}$ is $\mathcal{M}_2$, THEN $\tilde{u}(t) = \xi \mathcal{K}_2 \Im(t)$.We choose $\alpha = 0.91$, $\xi = 28$, $\hat{\xi} = 32$, $\eta = 7.2$, $\bar{\eta} = 5.7$. By solving (18)-(20), we obtain feedback gain matrices $\mathcal{K}_1$ and $\mathcal{K}_2$, observer gain matrices Ω_1 and Ω_2 as

$$ \mathcal{K}_1 = \mathcal{K}_2 = \begin{bmatrix} 0.0937 & 0.0385 & -0.0115 \\ -0.2005 & -0.0065 & -0.0304 \\ -0.1119 & 0.1005 & -0.0924 \end{bmatrix}, $$

$$ \Omega_1 = \Omega_2 = \begin{bmatrix} 0.0921 & 0.0823 \\ 0.0349 & -0.1197 \\ 0.1566 & 0.1117 \end{bmatrix}, $$

with $\vartheta = 9.6406 \ \hat{\vartheta} = 29.8760, \Theta_{\min} = 187$.

Thus FOMAS (3) and (4) expressed by T-S fuzzy model obtained the cyber-security consensus for each $k \in \mathscr{N}$. Fig. 4 depicts trajectories of FOMASs when DoS attacks happen in the communication network connecting the three layers without security control, where $\Im_p(t) = (\Im_{p1}(t), \Im_{p2}(t), \Im_{p3}(t))^T$, $p = 1, 2, \ldots, 5$. When $\omega_c = \frac{1}{4}\sum_{p=1}^4 |\omega_p|$, Fig. 5 denotes the consensus tracking error. When $\varpi_c = \frac{1}{4}\sum_{p=1}^4 |\varpi_p|$, the observer error of closed-loop systems is depicted in Fig. 6. Figs. 5 and 6 show that cyber attack occurs and the secure control mechanism still steers the system states to achieve consensus. Figs. 7 and 8 depict the comparison of the consensus tracking error ω_c, and observer error ϖ_c of systems of the different parameters ξ and $\hat{\xi}$. Figs. 9 and 10 show the comparison of the consensus tracking error ω_c, and observer error ϖ_c.

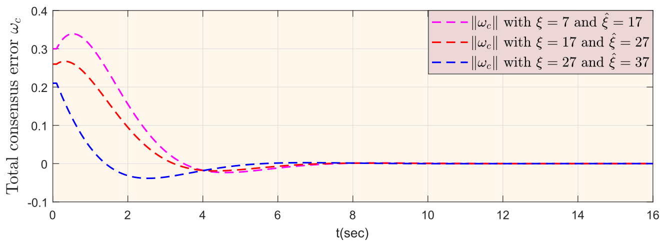

Fig. 7. Comparison of consensus errors ω_c, versus parameters ξ and $\hat{\xi}$ numerical simulations.

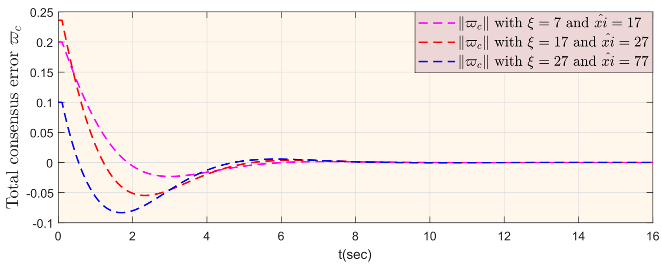

Fig. 8. Comparison of observer errors ϖ_c, versus parameters ξ and $\hat{\xi}$ numerical simulations.

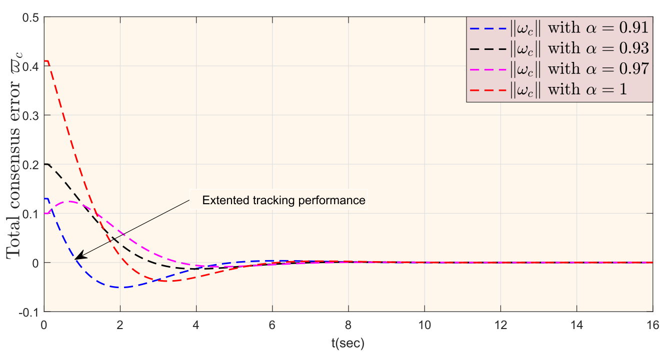

Fig. 9. Comparison of consensus errors ω_c, for differential orders. Notice that, for $\alpha = 0.91$ consensus errors performance effective manner when compared to order $\alpha = 1$.

Remark 3: Moreover, numerical simulations demonstrate that increasing the coupling strengths ξ and $\hat{\xi}$ improves both the convergence rates for consensus tracking and states observing (see Figs. 7 and 8 for details). This implies that, while the consensus tracking problem can be solved by adjusting the coupling strengths $\xi > \hat{\vartheta}/\hat{\zeta}, \hat{\zeta} = \frac{\lambda_{\min}(\Upsilon\mathscr{L}+\mathscr{L}\Upsilon)}{\gamma_{\max}}$ and $\hat{\xi} > \vartheta/\zeta$, $\zeta = \frac{\lambda_{\min}(\Upsilon\mathscr{L}+\mathscr{L}^T\Upsilon)}{\gamma_{\max}}$, the convergence rates may be quite small when the coupling strengths ξ and $\hat{\xi}$ are, respectively. Addition, another advantage for comparison tracking error ω_c, and observer error ϖ_c (see Figs. 9 and 10 for details). Notice that, for $\alpha = 0.91$, consensus errors and observer error performance effective manner when compared to order $\alpha = 1$. Briefly, according to the presented results, the T-S fuzzy FOMASs outperforms the secure control scheme exploiting integer-order operators.

*Example 2.*In this example, the suggested observer-based design approach is used to track the consensus of a network circuit for a tunnel diode model. We proposed the tunnel diode network circuit model as depicted in Fig. 11, where C_1, C_2 denote the capacitor, R_D represents the impedance of tunnel

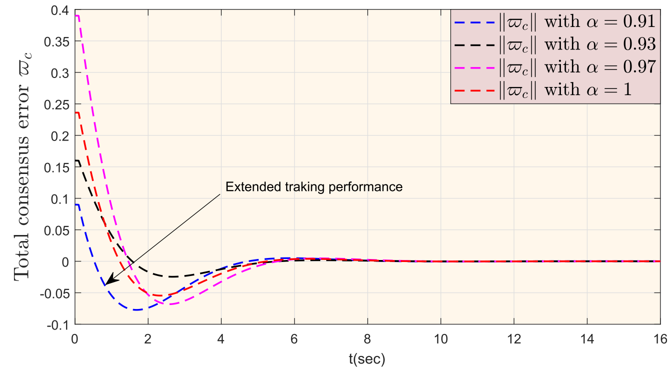

Fig. 10. Comparison of consensus errors ϖ_c, for differential orders. Notice that, for $\alpha = 0.91$ consensus errors performance effective manner when compared to order $\alpha = 1$.

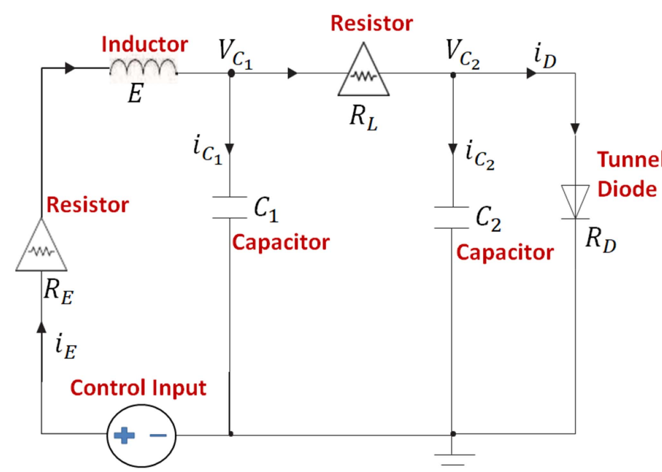

Fig. 11. The application network circuit of tunnel diode.

diode, E is inductor, and R_L, R_E are the linear resistance.The fractional-order calculus model is used to rewrite the dynamic description of the three-state tunnel diode network circuit model provided in the literature [46]:

$$\begin{cases} {}^{C}D_t^{\alpha}\Im(t) = \mathcal{A}\Im(t) + \mathcal{B}\tilde{u}(t), \\ \varphi(t) = \mathcal{C}\Im(t) \end{cases} \tag{40}$$

with

$$\mathcal{A} = \begin{bmatrix} -\frac{1}{R_L C_1} & \frac{1}{R_L C_1} & \frac{1}{C_1} \\ \frac{1}{R_L C_2} & -\frac{s_1 + s_2\Im_1^2(t)}{C_2} - \frac{1}{R_L C_2} & 0 \\ -\frac{1}{E} & 0 & -\frac{R_E}{E} \end{bmatrix},$$

$$\mathcal{B} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{E} \end{bmatrix}, \mathcal{C} = I, \Im(t) = (\Im_1, \Im_2, \Im_3)^T,$$

$$\Im_1(t) \in [m_1, m_2],\ m_1 = \max\{m_1^2, m_2^2\},$$

$$\textit{and } s_1, s_2 \text{ is a known scalars.}$$

The following equations can be expressed in T-S fuzzy form:

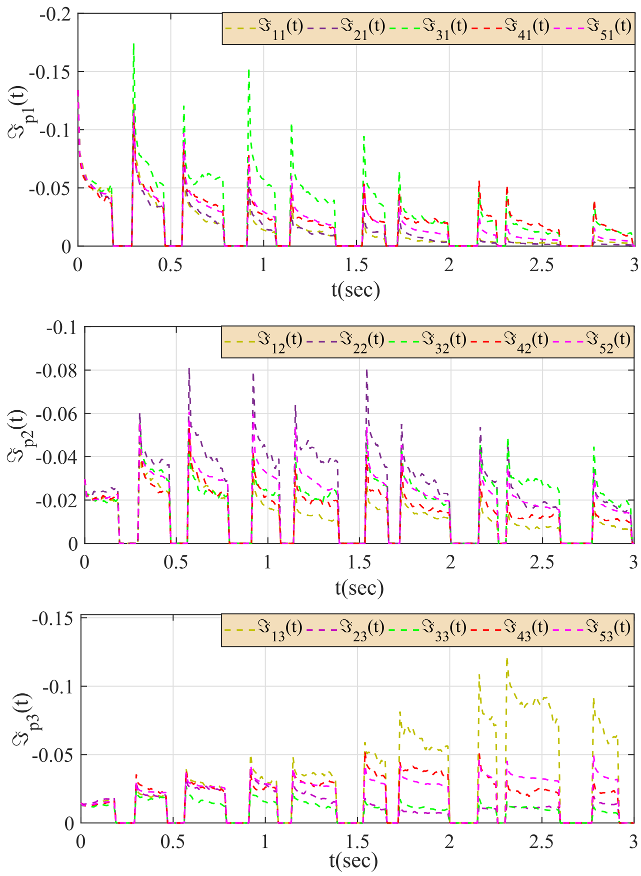

Fig. 12. State trajectories for tunnel diode circuit model when attacks occur with no security control, where $\Im_p(t) = (\Im_{p1}(t), \Im_{p2}(t), \Im_{p3}(t))^T$, $p = 1, 2, \ldots, 5$.

$$\begin{cases} {}_{t_0}^{\ C}D_t^{\alpha}\Im_p(t) = \sum_{\theta=1}^{2}\Psi_\theta(\phi_p(t))\big[\mathcal{A}_\theta\Im_p(t) + \mathcal{B}_\theta\tilde{u}_p(t)\big], \\ \varphi_p(t) = \sum_{\theta=1}^{2}\Psi_\theta(\phi_p(t))\big[\mathcal{C}_\theta\Im_p(t)\big], \end{cases} \tag{41}$$

where

$$\Im_p(t) = [\Im_{p1}, \Im_{p2}, \Im_{p3}]^T,$$

$$\mathcal{A}_1 = \begin{bmatrix} -\frac{1}{R_L C_1} & \frac{1}{R_L C_1} & \frac{1}{C_1} \\ \frac{1}{R_L C_2} & -\frac{s_1+s_2 m_1}{C_2} - \frac{1}{R_L C_2} & 0 \\ -\frac{1}{E} & 0 & -\frac{R_E}{E} \end{bmatrix},$$

$$\mathcal{A}_2 = \begin{bmatrix} -\frac{1}{R_L C_1} & \frac{1}{R_L C_1} & \frac{1}{C_1} \\ \frac{1}{R_L C_2} & -\frac{s_1}{C_2} - \frac{1}{R_L C_2} & 0 \\ -\frac{1}{E} & 0 & -\frac{R_E}{E} \end{bmatrix},$$

$$\mathcal{B}_1 = \mathcal{B}_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{E} \end{bmatrix}, \ \mathcal{C} = I,$$

$$\mathscr{M}_1 = \frac{\Im_{p1}^2(t)}{m_1}, \mathscr{M}_2 = 1 - \frac{\Im_{p1}^2(t)}{m_1}, \Im_{p1} \in [-4, 4].$$

Here, the order α is chosen as 0.93, and similar to [46], the values of s_1, s_2, C_1, C_2, E, R_E, R_L are selected as 0.002, 0.01, 1F, 0.1F, 20H, 0Ω and 1Ω respectively. Select one tunnel diode network circuit model to act as the leader agent, generating the required state trajectory. With the network topology shown in Fig. 3, four tunnel diode circuit models operate as follower agents, and these agents can receive output information from their neighbours. Moreover, let $h = 0.02\ s$ and $t_p = 90\ ph$, $\bar{t}_p = 91\ ph$, for $p \in \mathscr{N}$. Assume that attacks occur over the time interval $[t_p, \bar{t}_p]$. We choose $\xi = 13$, $\hat{\xi} = 19$, $\eta = 3.7$, $\bar{\eta} = 7.9$. By solving (18)-(20), we obtain feedback gain matrices $\mathcal{K}_1$ and $\mathcal{K}_2$, observer gain matrices Ω_1 and Ω_2 as

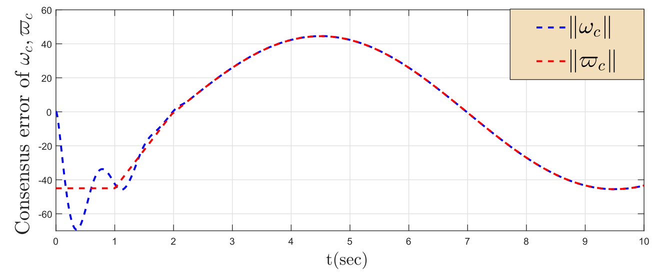

Fig. 13. Consensus error ω_c and ϖ_c, when attacks occur with no security control.

$$\mathcal{K}_1 = \begin{bmatrix} -0.0794 & -0.0008 & 0 \\ 0.0008 & -0.0734 & 0 \\ 0 & 0 & -0.0974 \end{bmatrix},$$

$$\mathcal{K}_2 = \begin{bmatrix} 2.3741 & 0.1895 & 0 \\ 0 & 1.3604 & -0.9087 \\ 0.1895 & -0.9087 & 0.3761 \end{bmatrix},$$

$$\Omega_1 = \begin{bmatrix} -2.9031 & -0.7980 \\ 3.2080 & 0.3950 \\ -1.7935 & -0.9705 \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} -1.9464 & 0.4488 \\ 0.8713 & -0.8736 \\ -1.3688 & -0.9125 \end{bmatrix}$$

with $\vartheta = 39.0318$, $\hat{\vartheta} = 38.4191$, $\Theta_{\min} = 374$. When no security controls are used and attacks occur at $t_1 = 1.4\ s$, the state trajectories of the 5 agents are represented in Fig. 12. The final tracking to the leader was not possible due to difficulties in the connectivity of the two communication networks. Fig. 13 shows that without secure control scheme of system model is still not achieve consensus error. As a result, to evaluate the distributed observer-based security control mechanism, the followers are implemented, and the consensus errors are presented in Fig. 14, indicating that good observation performance has been obtained.

V. Conclusion

The security control for T-S fuzzy FOMASs with cyber-attacks has been examined. A cyber-attack model with malicious attacks are considered with both controllers and observers. For modeling recoverable cyber attacks a switched device has been employed. By utilizing the theory of fractional-

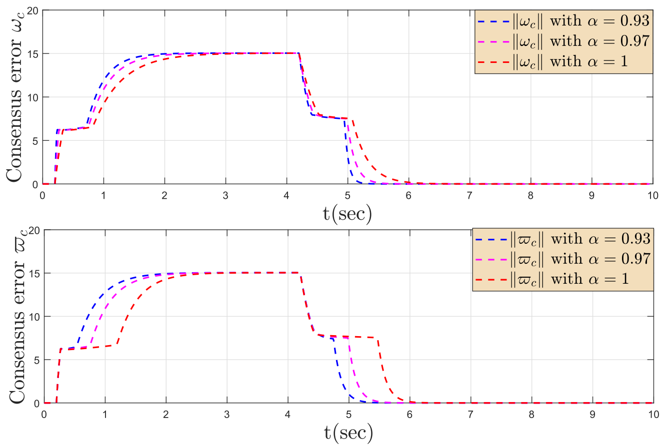

Fig. 14. Consensus errors ω_c and ϖ_c with different differential orders.

calculus, Lyapunov functional and algebraic graph theory, an distributed control is designed to achieve the secure consensus of T-S fuzzy FOMASs. Finally, a simulation examples and an electronic circuit based on a tunnel diode are presented to demonstrate the effectiveness of the suggested strategy. In the future, we will investigate the networking of T-S fuzzy cascade multi-area power systems in a smart grid with electric vehicles (EVs) under DoS attacks.

REFERENCES

[1] Y. Tang, X. Xing, H. Karimi, and L. Kocarev, "Tracking control of networked multi agent systems under new characterizations of impulses and its applications in robotic systems," *IEEE Trans. Ind. Electron.*, vol. 63, pp. 1299–1307, Feb. 2016.

[2] C. Tomlin, G. Pappas, and S. Sastry, "Conflict resolution for air traffic management: A study in multi-agent hybrid systems," *IEEE Trans. Autom. Control.*, vol. 43, pp. 509–521, Apr. 1998.

[3] R. W. Beard, T. W. McLain, M. A. Goodrich, and E. P. Anderson, "Coordinated target assignment and intercept for unmanned air vehicles," *IEEE Trans. Robot. Autom.*, vol. 18, pp. 911–92, Dec. 2002.

[4] C. Dou, D. Yue, J. M. Guerrero, X. Xie, and S. Hu, "Multi-agent system-based distributed coordinated control for radial dc micro grid considering transmission time delays," *IEEE Trans. Smart. Grid.*, vol. 8, pp. 2370–2381, Sep. 2017.

[5] Y. Zheng, J. Ma, and L. Wang, "Consensus of hybrid multi-agent systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, pp. 1359–1365, Apr. 2018.

[6] W. Zou, P. Shi, Z. Xiang, and Y. Shi, "Finite-time consensus of second-order switched nonlinear multi-agent systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, pp. 1757–1762, May 2020.

[7] G. Wen, W. Yu, Y. Xia, X. Yu, and J. Hu, "Distributed tracking of nonlinear multi-agent systems under directed switching topology: An observer-based protocol," *IEEE Trans. Syst. Man Cybern.*, vol. 47, pp. 869–881, 2017.

[8] S. Liu, Z. Wang, G. Wei, and M. Li, "Distributed set-membership filtering for multirate systems under the round-robin scheduling over sensor networks," *IEEE Trans. Cybern.*, vol. 50, pp. 1910–1920, 2020.

[9] M. Syed Ali, R. Agalya, V. Shekher, and Y. H. Joo, "Non-fragile sampled data control for stabilization of non-linear multi-agent system with additive time varying delays, markovian jump and uncertain parameters," *Nonlinear Anal.: Hybrid Syst.*, vol. 36, 2020, Art. no. 100830.

[10] P. Gong and W. Lan, "Adaptive robust tracking control for multiple unknown fractional-order nonlinear systems," *IEEE Trans. Cybern.*, vol. 49, pp. 1365–1376, 2019.

[11] P. Mani, R. Rajan, L. Shanmugam, and Y. H. Joo, "Adaptive fractional fuzzy integral sliding mode control for PMSM model," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 8, pp. 1674–1686, Aug. 2019.

[12] F. Wang, Y. Yang, A. Hu, and X. Xu, "Exponential synchronization of fractional-order complex networks via pinning impulsive control," *Nonlinear Dyn.*, vol. 82, pp. 1979–1987, 2015.

[13] S. Zhang, Y. Yu, and J. Yu, "LMI conditions for global stability of fractional-order neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 10, pp. 2423–2433, Oct. 2017.

[14] M. Syed Ali, G. Narayanan, V. Shekher, A. Alsaedi, and B. Ahmad, "Global mittag-leffler stability analysis of impulsive fractional-order complex-valued BAM neural networks with time varying delays," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 83, 2020, Art. no. 105088.

[15] Y. Cao, Y. Li, W. Ren, and Y. Chen, "Distributed coordinative of networked fractional-order systems," *IEEE Trans. Syst. Man Cybern. Part B., Cybern.*, vol. 40, no. 2, pp. 362–370, Apr. 2010.

[16] H. Liu, L. Cheng, M. Tan, and Z. G. Hou, "Exponential finite-time consensus of fractional-order multiagent systems," *IEEE Trans. Syst. Man Cybern. B. Cybern.*, vol. 50, no. 4, pp. 1549–1558, Apr. 2020.

[17] J. Chen, B. Chen, and Z. Zeng, "Synchronization and consensus in networks of linear fractional-order multi-agent systems via sampled-data control," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 8, pp. 2955–2964, Aug. 2020.

[18] Z. Li, L. Gao, W. Chen, and Y. Xu, "Distributed adaptive cooperative tracking of uncertain nonlinear fractional-order multi-agent systems," *IEEE-CAA J. Autom..*, vol. 7, no. 1, pp. 292–300, Jan. 2020.

[19] G. Ren, Y. Yu, C. Xu, and X. Hai, "Consensus of fractional multi-agent systems by distributed event-triggered strategy," *Nonlinear Dyn.* vol. 95, pp. 541–555, 2019.

[20] Y. Li, Y. Liu, and S. Tong, "Observer-based neuro-adaptive optimized control of strict-feedback nonlinear systems with state constraints," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 33, no. 7, pp. 3131–3145, Jul. 2022, doi: 10.1109/TNNLS.2021.3051030.

[21] Y. Yang, H. Xu, and D. Yue, "Observer-based distributed secure consensus control of a class of linear multi-agent systems subject to random attacks," *IEEE Trans. Circuits Syst. I., Regular Papers.*, vol. 66, no. 8, pp. 3089–3099, Aug. 2019.

[22] L. Wang, Z. Wang, G. Wei, and F. E. Alsaadi, "Observer-based consensus control for discrete-time multi agent systems with coding-decoding communication protocol," *IEEE Trans. Cybern.*, vol. 49, no. 12, pp. 4335–4345, Dec. 2019.

[23] Y. Wan, J. Cao, G. Chen, and W. Huang, "Distributed observer-based cyber-security control of complex dynamical networks," *IEEE Trans. Circuits Syst. I., Reg. Papers.*, vol. 64, no. 11, pp. 2966–2975, Nov. 2017.

[24] W. Yu, Y. Li, G. Wen, X. Yu, and J. Cao, "Observer design for tracking consensus in second-order multi-agent systems: Fractional order less than two," *IEEE Trans. Autom. Control.*, vol. 62, no. 2, pp. 894–900, Feb. 2017.

[25] Y. Gong, G. Wen, Z. Peng, T. Huang, and Y. Chen, "Observer-based time-varying formation control of fractional-order multi-agent systems with general linear dynamics," *IEEE Trans. Circuits Syst., II, Exp. Briefs*, vol. 67, no. 1, pp. 82–86, Jan. 2020.

[26] H. Yang and D. Ye, "Observer-based fixed-time secure tracking consensus for networked high-order multi-agent systems against DoS attacks," *IEEE Trans. Cybern.*, vol. 52, no. 4, pp. 2018–2031, Apr. 2022, doi: 10.1109/TCYB.2020.3005354.

[27] D. Ding, Z. Wang, D. W. C. Ho, and G. Wei, "Observer-based event-triggering consensus control for multiagent systems with lossy sensors and cyber-attacks," *IEEE Trans. Cybern.*, vol. 47, no. 8, pp. 1936–1947, Aug. 2017.

[28] H. Sandberg, S. Amin, and K. Johansson, "Cyberphysical security in networked control systems: An introduction to the issue," *IEEE Control Syst.*, vol. 35, no. 1, pp. 20–23, Feb. 2015.

[29] D. Ding, Q. L. Han, X. Ge, and J. Wang, "Secure state estimation and control of cyber-physical systems: A survey," *IEEE Trans. Syst. Man Cybern.*, vol. 51, no. 1, pp. 176–190, Jan. 2021.

[30] H. Song, D. Ding, H. Dong, and X. Yi, "Distributed filtering based on cauchy-kernel-based maximum correntropy subject to randomly occurring cyber-attacks," *Automatica*, vol. 135, 2022, Art. no. 110004.

[31] J. Liu, T. Yin, D. Yue, H. R. Karimi, and J. Cao, "Event-based secure leader-following consensus control for multiagent systems with multiple cyber attacks," *IEEE Trans. Cybern.*, vol. 51, no. 1, pp. 162–173, Jan. 2021, doi: 10.1109/TCYB.2020.2970556.

[32] Y. Xu, M. Fang, Z. Wu, Y. Pan, M. Chadli, and T. Huang, "Input-based event-triggering consensus of multiagent systems under denial-of-service attacks," *IEEE Trans. Syst. Man Cybern.*, vol. 50, no. 4, pp. 1455–1464, Apr. 2020.

[33] Y. Wan, G. Wen, X. Yu, and T. Huang, "Distributed consensus tracking of networked agent systems under denial-of-service attacks," *IEEE Trans. Syst. Man Cybern.*, vol. 51, no. 10, pp. 6183–6196, Oct. 2021.

[34] W. L. He, X. Y. Gao, W. M. Zhong, and F. Qian, "Secure impulsive synchronization control of multi-agent systems under deception attacks," *Inf. Sci.*, vol. 459, pp. 354–68, 2018.

[35] K. Shi, J. wang, S. Zhong, Y. Tang, and J. Cheng, "Hybrid-driven finite-time H_∞ sampling synchronization control for coupling memory complex networks with stochastic cyber attacks," *Neurocomputing*, vol. 387, pp. 241–254, 2020.

[36] Z. Feng, G. Hu, and G. Wen, "Distributed consensus tracking for multi-agent systems under two types of attacks," *Int. J. Robust Nonlinear Control*, vol. 26, pp. 896–918, 2016.

[37] M. M. Hossain and C. Peng, "Observer-based event triggering H_∞ LFC for multi-area power systems under DoS attacks," *Inf. Sci.*, vol. 543, pp. 437–453, 2021.

[38] H. Shen, Y. Men, Z. Wu, and J. H. Park, "Nonfragile H_∞ control for fuzzy markovian jump systems under fast sampling singular perturbation," *IEEE Trans. Syst. Man Cybern.*, vol. 48, no. 12, pp. 2058–2069, Dec. 2018.

[39] Y. Zhao, B. Li, J. Qin, H. Gao, and H. R. Karimi, "H_∞ consensus and synchronization of nonlinear systems based on a novel fuzzy model," *IEEE Trans. Cybern.*, vol. 43, no. 6, pp. 2157–2169, Dec. 2013.

[40] Q. Shen, B. Jiang, P. Shi, and J. Zhao, "Cooperative adaptive fuzzy tracking control for networked unknown nonlinear multiagent systems with time-varying actuator faults," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 3, pp. 494–504, Jun. 2014.

[41] Y. Ma and J. Zhao, "Cooperative output regulation for nonlinear multi-agent systems described by T-s fuzzy models under jointly connected switching topology," *Neurocomputing*, vol. 332, pp. 351–359, 2019.

[42] J. H. Moon and H. J. Lee, "Graph laplacian-based leaderless consensus for multi-agent systems in T-s form," *Fuzzy Set Syst.*, vol. 368, pp. 119–136, 2019.

[43] Y. Cheng, T. Hu, Y. Li, and S. Zhong, "Consensus of fractional-order multi-agent systems with uncertain topological structure: A. takagi-sugeno fuzzy event-triggered control strategy," *Fuzzy Sets Syst.*, vol. 416, pp. 64–85, 2021.

[44] J. Liu, T. Yin, J. Cao, D. Yue, and H. R. Karimi, "Security control for TS fuzzy systems with adaptive event-triggered mechanism and multiple cyber-attacks," *IEEE Trans. Syst. Man Cybern.*, vol. 51, no. 10, pp. 6544–6554, Oct. 2021.

[45] J. Liu, L. Wei, X. Xie, E. Tian, and S. Fei, "Quantized stabilization for TS fuzzy systems with hybrid-triggered mechanism and stochastic cyber-attacks," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3820–3834, Dec. 2018.

[46] J. Liu, M. Yang, E. Tian, and J. Cao, "Event-based security control for state-dependent uncertain systems under hybrid-attacks and its application to electronic circuits," *IEEE Trans. Circuits Syst. I Regular Pap.*, vol. 66, no. 12, pp. 4817–4828, Dec. 2019.

G. Narayanan was born in 1994. He received the B. Sc. degree in the the field of mathematics from C. Abdul Hakeem College, Melvisharam, Vellore, affiliated to Thiruvalluvar University, Vellore, India, in 2016, the Post Graduation and the Ph.D. degrees in mathematics from Thiruvalluvar University, Vellore, India, in 2018 and 2022, respectively, and the Ph.D. degree under the guidance of Dr. M. Syed Ali. He is currently an assistant professor of mathematics with the Center for Computational Modeling, Chennai Institute of Technology, Chennai, India. He has authored and coauthored of many research articles in various SCI journals. His research interests include fractional-order nonlinear systems, wind turbine system, cyber security control, complex-valued & quaternion-valued of molecular systems, and multi-agent networked systems. He is also a Reviewer for various SCI journals.

M. Syed Ali received the Graduation degree from the Department of Mathematics, Gobi Arts and Science College and the Postgraduation degree in mathematics from the Sri Ramakrishna Mission Vidyalaya College of Arts and Science, affiliated to Bharathiar University, Coimbatore, India, in 2002 and 2005, respectively. He was selected as a Postdoctoral Fellow in the year 2010 for promoting his research in the field of mathematics, Bharathidasan University, Trichy, India, and also worked there from November 2010 to February 2011. Since March 2011, he has been an Assistant Professor with the Department of Mathematics, Thiruvalluvar University, Vellore, India. He has authored or coauthored more than 190 Research Papers in various SCI journals holding impact factors and Research articles in national journals and international conference proceedings. He is also a reviewer for several SCI journals. His research interests include stochastic differential equations, dynamical systems, fuzzy neural networks, complex networks and cryptography. He was the recipient of the Master of philosophy in 2006 in the field of mathematics with specialised area of numerical analysis from Gandhigram Rural University, Gandhigram, India, the Doctor of philosophy in 2010 in the field of mathematics specialised in the area of fuzzy neural networks in Gandhi Gram Rural University, Gandhigram, India, and the Young Scientist Award 2016 by The Academy of Sciences, Chennai. Dr M. Syed Ali has been listed in 2% of World Scientists as released by Stanford University, Stanford, CA, USA, on 2020, 2021, and 2022, respectively.

Quanxin Zhu (Senior Member, IEEE) received the Ph.D. degree from Sun Yat-sen (Zhongshan) University, Guangzhou, China, in 2005. He is currently a Professor with Hunan Normal University, Changsha, China. He is a Highly Cited Scientist in the world, in 2018 and 2020, respectively. He has authored or coauthored more than 200 journal papers. His research interests include stochastic control, stochastic delayed systems, stochastic stability, nonlinear systems, Markovian jump systems, and stochastic complex networks. He was the recipient of the Alexander von Humboldt Foundation of Germany.

Bandana Priya is currently a Professoor of mathematics with the Department of Applied Sciences, G L Bajaj Institute of Technology and Management, Greater Noida, India. She is equipped with an extraordinary caliber and appreciable academic potency. She has authored or coauthored many research paper in journals of high repute. Her research interests include optimization technique, fuzzy graph theory, and mathematical control theory.

Ganesh Kumar Thakur received the M.Sc. degree in mathematics by securing the first rank and the Ph.D. degree in mathematics from Tilka Manjhi Bhagalpur University, Bhagalpu, India. He has been associated with ABES Enginering College, Ghaziabad, India, as a competent Associate Professor of mathematics, Department of Applied Science. As an ardent Researcher, he has authored or coauthored 25 research papers in reputed referred journals. He has also Coguided one research Scholar. He is the Member of various Mathematical Societies.